## QUESTION 1 – INDUCTION PROOF

It is difficult to prove the statement in its current form, since the statement says that the second function’s arguments must be size’ m 0.

However, after one iteration the second function’s arguments will be size’ m x, where x != 0 (as 1 + acc is passed as an argument).

So a more useful generalized statement to prove might be:

(size m) + acc = size’ m acc

Base Case

The base case is for a mobile with just an object, ie Obj w.

size Obj w + acc

size Obj w = 1 //From program

size Obj w + acc = 1 + acc

size’ Obj w acc = acc + 1 = 1+acc //From program

Hence in the base case where m = Obj w,

(size m) + acc = size’ m acc

Inductive Hypothesis

As the inductive hypothesis, let us assume that for 2 arbitrary mobiles m1, m2 , that

(size m1) + acc = size’ m1 acc,

and that (size m2) + acc = size’ m2 acc.

Since acc is arbitrary, we can also say that the above statements hold when acc = 0, ie that size m1 = size’ m1 0.

Step Case

Now we can consider a mobile Wire(m1,m2). We need to prove that (size Wire(m1,m2)) + acc = size’ Wire(m1,m2) acc.

size’ Wire(m1,m2) acc =

size’ m1 (size’ m2 (1+ acc)) //From program

= size’ m1 (size m2 + (1 + acc)) //From i.h, using acc as 1+acc

= size m1 + (size m2 + (1+acc)) //From i.h, using acc as (size m2 + (1+acc))

= 1 + acc + size m1 + size m2

= (size Wire(m1,m2)) + acc

Thus proving the left hand size of the statement. Since the statement was generalized for an arbitrary acc, it means that the inductive hypotheses could be used even for values such as acc = (size m2 + (1+acc)).